Interface Simulation Methods

Introduction

To solve the color equation representing the evolution of an interface between two phases without numerical artificial diffusion or dispersion, two types of methods have been introduced in the recent three decades: geometrical interface-reconstructing methods and methods based on algebraically formulated differencing schemes. The geometric methods, consisting of several methods such as Simple Line Interface Calculation (SLIC) method, Piecewise Linear Interface Calculation (PLIC), Youngs's method and so on (reviewed by Scardoveli 1999), reconstruct the free surface with pricewise lines and then calculate the convection equation based on those reconstructions. These methods have complex free surface re-construction in 3D. On the other hand, the algebraic methods are easier to implement, some of these methods are possible to apply to the computation over BFC grids. With the cell face flux computed by a high order and a low order scheme, Zalesak (1979) proposed the flux corrected transport algorithm to predict the convection equation with a limiter to avoid numerical diffusion. Based on the high-resolution scheme, Ubbink et al. (1997 & 1999) proposed the most popular CICSAM scheme for the free surface flow simulation over the unstructured meshes. Other schemes based on the same ideas have derived then after such as HRIC (Muzaferija & al. 1998), Modified HRIC (Park & al 2009) and STACS (Darwich & al 2006). Wacławczyk, T.& al (2008) give a detailed description of the CICSAM and HRIC scheme as well as their comparison on specific test cases.

Normalized Variable Diagram

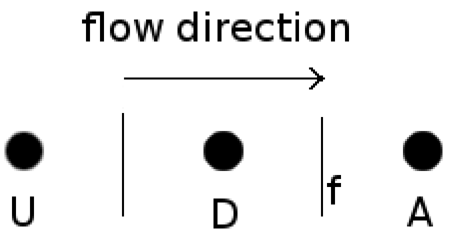

Figure 1: Schematic view of the donor-acceptor scheme.

The NVD by Leonard (1991) provides the foundation for the high order schemes CICSAM, HRIC, modified HRIC and STACS. Based on the notation introduced in Figure 1, and the Convective Boundedness Criterion (CBC) $\varphi_D \leq \varphi_f \leq \varphi_A$ for a variable φ on the cell face f, normalized variables are introduced

$$\tilde{\phi}_f = \frac{\phi_f - \phi_U}{\phi_A - \phi_U}, \quad \tilde{\phi}_D = \frac{\phi_D - \phi_U}{\phi_A - \phi_U} \qquad (1)$$

From here it follows that if φ_f is a function of φ_D, φ_A and φ_U, then the normalized variable $\tilde{\phi}_f$ is only a function of $\tilde{\phi}_D$ (since $\tilde{\phi}_A$ = 1 and $\tilde{\phi}_U$ = 0. This is the basis of the normalized variable diagram, which is a plot of the functional relationship between the normalized convected face value $\tilde{\phi}_f$ and the normalized adjacent donor node value $\tilde{\phi}_D$.

Some of the findings made by Leonard are summarized in Figure 2. It has been shown that NVD characteristics which pass through the second quadrant produces oscillations. It has also been found that the characteristics which pass through the fourth quadrant, i.e., below O are artificially diffusive. Numerical experimentation has shown that NVD characteristics that pass above P are oscillatory and characteristics that pass below P are artificially diffusive. Thus, to avoid oscillations without being artificially diffusive, the desired NVD characteristic should pass through O and P. This is depicted in the upper part of Figure 2.

The right side of the same Figure shows how the NVD can be utilized in practice. Any scheme that falls outside of the shaded triangle will be unbounded. Schemes near the upper bound will be more compressive, while schemes nearer the upwind scheme will be more diffusive. Thus, the NVD can be used to evaluate the degree of boundedness and diffusiveness of a scheme without actually implementing it.

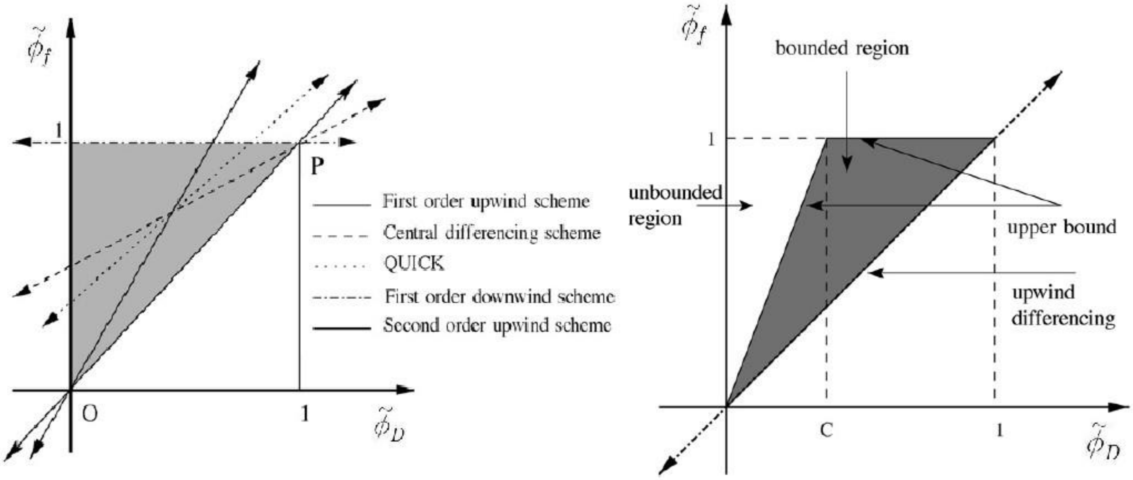

Figure 2: Overview of the NVD. Modified after [Leonard (1991)]

CICSAM: Compressive Interface Capturing Scheme for Arbitrary Meshes

The Compressive Interface Capturing Scheme for Arbitrary Meshes (CICSAM) scheme was presented by Ubbink in 1997. The scheme is a blend between the Hyper-C and the ULTIMATE-QUICKEST (UQ) schemes, with a blending factor of γ_f based on the angle between the interface and the direction of motion. As the name suggest, this scheme can be applied to arbitrary meshes. The Hyper-C scheme is defined as follows

$$\tilde{\phi}_{fCBC} = \begin{cases} \min\left(1, \frac{\tilde{\phi}_D}{C_f}\right) & for \quad 0 \le \tilde{\phi}_D \le 1 \\ \tilde{\phi}_D \quad for\ \tilde{\phi}_D \notin [0,1] \end{cases} \qquad (2)$$

The scheme satisfies the CBC criterion, and is compressive to its nature. This compressive feature may in some situations deform the shape of the interface and the scheme is therefore blended with a less diffusive UQ scheme. The UQ scheme has been derived from the QUICK scheme and is defined as follows:

$$\tilde{\phi}_{fUQ} = \begin{cases} \min\left(\frac{8C_f\tilde{\phi}_D + (1-C_f)(6\tilde{\phi}_D + 3)}{8}, \tilde{\phi}_{fCBC}\right) & for \quad 0 \le \tilde{\phi}_D \le 1 \\ \tilde{\phi}_D \quad for\ \tilde{\phi}_D \notin [0,1] \end{cases} \qquad (3)$$

A smooth blend between the Hyper-C and UQ schemes is ensured by the blending factor $0 \le \gamma_f \le 1$ as follows:

$$\tilde{\phi}_f = \gamma_f \tilde{\phi}_{fCBC} + (1-\gamma_f)\tilde{\phi}_{fUQ} \tag{4}$$

Where

$$\gamma_f = \min\left(\frac{1+\cos 2\theta_f}{2}, 1\right) \tag{5}$$

$$\vartheta_f = \arccos|dn|, \tag{6}$$

where **n** is the normal vector of the interface and **d** is the vector parallel to the line between the centers of the donor and acceptor cells. The NVD of the CICSAM scheme is presented in Figure 3.

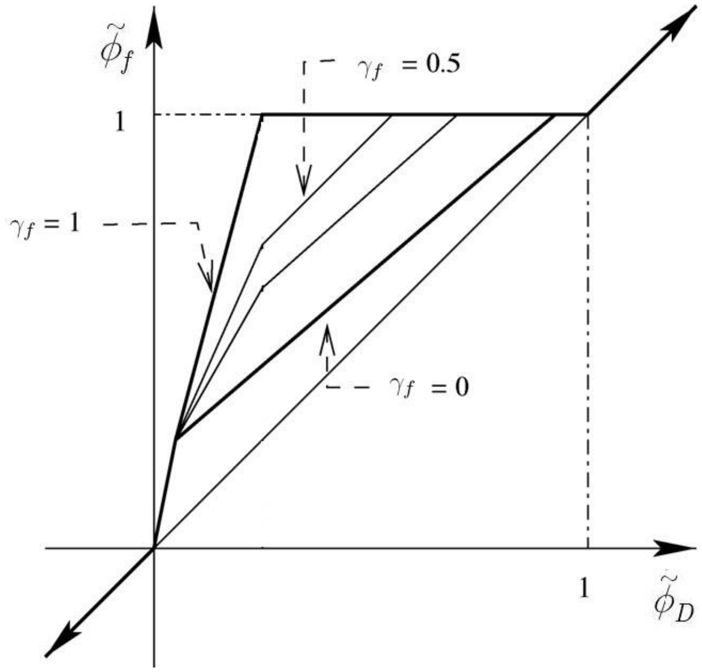

Figure 3: NVD for the CICSAM scheme.

HRIC: High Resolution Interface Capturing Scheme

The HRIC scheme was introduced by Muzaferija in 1998 with the intention of simplifying the CICSAM method and to avoid the explicit dependence on the CFL condition. This method is based on blending the bounded downwind and upwind differencing schemes. The aim being a combination of the compressive properties of the bounded downwind scheme, with the stability of the upwind differencing scheme. The normalized cell face value in one dimension will be

$$\tilde{\phi}_f = \begin{cases} 2\tilde{\phi}_D, & \tilde{\phi}_D \in [0, 0.5] \\ 1, & \tilde{\phi}_D \in [0.5, 1] \\ \tilde{\phi}_D, & \tilde{\phi}_D \notin [0, 1] \end{cases} \tag{7}$$

In order to satisfy the CBC, first order upwind differencing scheme is used. Again, a smooth blending is achieved by the blending factor γ_f connected with angle ϑ_f

$$\tilde{\phi}_f^* = \gamma_f \tilde{\phi}_f + \tilde{\phi}_D (1-\gamma_f), \quad (8)$$

where $\gamma_f = \sqrt{|\cos(\theta_f)|}$, and ϑ_f is the angle between the interface normal and the vector parallel to the line between the centers of the donor and acceptor cells.

In order to avoid stability problems when the CFL condition is not satisfied, $\tilde{\phi}_f^*$ is corrected with respect to the local Courant number c_f. This will ensure a continuous switching between the schemes in the time domain as well.

$$\tilde{\phi}_f^{**} = \begin{cases} \tilde{\phi}_f^*, & for\, C_f < 0.3 \\ \tilde{\phi}_D + \left(\tilde{\phi}_f^* - \tilde{\phi}_D\right)\dfrac{0.7 - C_f}{0.7 - 0.3}, & for\, 0.3 \le C_f \le 0.7 \\ \tilde{\phi}_D, & for\, C_f > 0.7 \end{cases} \quad (9)$$

The NVD for HRIC at different Courant numbers is depicted in Figure 4.

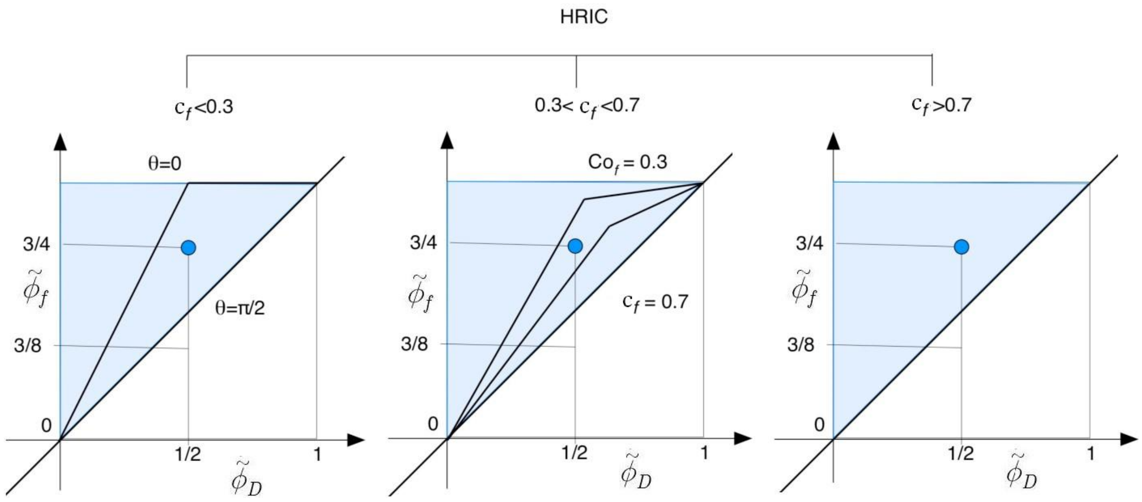

Figure 4: NVD for the HRIC scheme at different Courant numbers.

Modified HRIC (MHRIC) (Park et al.2009)

Let:

$$\tilde{\phi}_f = \begin{cases} 2\tilde{\phi}_D, & \tilde{\phi}_D \in [0,0.5] \\ 1, & \tilde{\phi}_D \in [0.5,1] \\ \tilde{\phi}_D, & \tilde{\phi}_D \notin [0,1] \end{cases} \quad \text{and} \quad \tilde{\phi}_f^* = \begin{cases} \min\left(\dfrac{6\tilde{\phi}_D + 3}{8}, \tilde{\phi}_f\right), & \tilde{\phi}_D \in [0,1] \\ \tilde{\phi}_D, & \tilde{\phi}_D \notin [0,1] \end{cases} \quad (10)$$

MHRIC can be written as:

$$\tilde{\phi}_f^{**} = \gamma_f \tilde{\phi}_f + (1-\gamma_f)\tilde{\phi}_f^* \quad (11)$$

With $\gamma_f = \sqrt{|\cos(\theta_f)|}$. The NVD of the scheme and the CBC region are shown in figure 5.

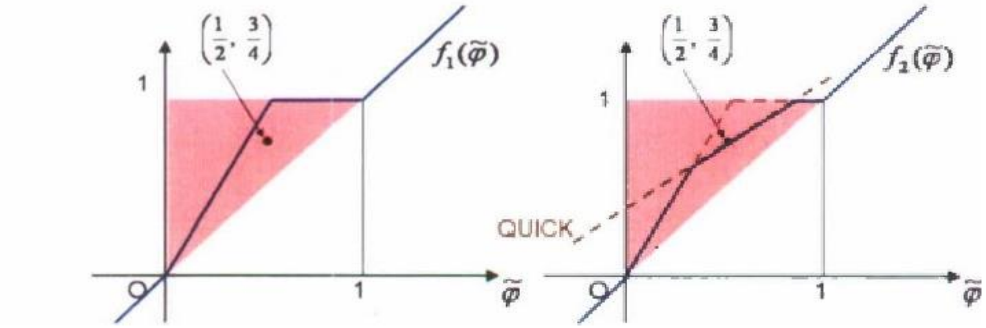

Figure 5. NVD of MHRIC scheme with CBC

Switching Technique for Advection and Capturing of Surfaces (STACS) method

One of the drawbacks of HRIC and CICSAM schemes is high Courant numbers. Both methods lack a proper switching strategy to accurately model the interface when Courant number increases. STACS method has been proposed to improve the accuracy and stability of the results specifically in high Courant numbers by Darwish & al (2006). It uses an implicit transient discretization, i.e. no transient bounding is applied, and in order to minimize the stepping behavior of HRIC scheme, a modification is proposed. In this method, applying $\gamma_f = \cos^4(\theta_f)$ term is designed instead of $\gamma_f = \sqrt{|\cos(\theta_f)|}$ in Eq. 13. The STACS method is:

$$\tilde{\phi}_{fSUP} = \begin{cases} \tilde{\phi}_D & for\ \tilde{\phi}_D \notin]0,1[\\ 2\tilde{\phi}_D & for\ 0 < \tilde{\phi}_D \le \frac{1}{3} \\ \frac{1+\tilde{\phi}_D}{2} & for\ \frac{1}{3} < \tilde{\phi}_D \le \frac{1}{2} \\ \frac{3\tilde{\phi}_D}{2} & for\ \frac{1}{2} < \tilde{\phi}_D \le \frac{2}{3} \\ 1 & for\ \frac{2}{3} < \tilde{\phi}_D < 1 \end{cases} \quad \text{and} \quad \tilde{\phi}_{fSTOIC} = \begin{cases} \tilde{\phi}_D & for\ \tilde{\phi}_D \notin]0,1[\\ \frac{1+\tilde{\phi}_D}{2} & for\ 0 < \tilde{\phi}_D \le \frac{1}{2} \\ \frac{3+6\tilde{\phi}_D}{8} & for\ \frac{1}{2} < \tilde{\phi}_D \le \frac{5}{6} \\ 1 & for\ \frac{5}{6} < \tilde{\phi}_D < 1 \end{cases} \tag{12}$$

$$\tilde{\phi}_f = \gamma_f \tilde{\phi}_{fSUP} + (1-\gamma_f)\tilde{\phi}_{fSTOIC} \tag{13}$$

This enables a rapid but smooth switching strategy that works very well, especially where the normal to the free-surface face is not along the grid direction.

The choice of these methods is dependent on the value of the Courant number. For Courant number, smaller than 0.3 CICSAM should be preferred. For values of Courant numbers below 0.5 and above 0.3, the HRIC and modified HRIC could be used. For higher values of Courant number, the STACS and MHRIC should be preferred. For large domain (like flow around ships), HRIC and MHRIC should be preferred.

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