Two-phase numerical model of heat transfer in a natural basin

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Abstract

The state of two-phase flow 'liquid-gas' has been modeled numerically by the three-dimensional method of complex research of heat and mass transfer. This allows examining the interaction of some transfer processes in the natural cooling basin (lake Drūkšiai): power and direction of the wind, variable density of the water, heat conduction, direct and diffusive solar radiation, radiative exchange, evaporation cooling function factor, friction and heat transfer coefficient of the water-air interface. The combined effect of these natural actions determines the heat amount that the basin is able to dissipate to the surrounding atmospheric media in thermal equilibrium.

This article presents a number of most widely used expressions for vertical and horizontal heat transfer coefficients. Basing on the stream velocity and mean temperature profiles measured in the cooling pond, as well as on their time variations, suggestions are made that the mixing rate at the water surface is caused by natural space – time variation of the wind, and can be described by the value of eddy viscosity coefficient – 6.7 m^2/s at mean wind 5.0 m/s.

The wind influences the surface of the lake according experimental data, i. e. 1 – 3% of the mean wind velocity. Model is applying for weakly wind, approximately 1 – 5 m/s of the mean wind velocity.

A comparison experimental and numerical result showed a qualitative agreement. Simultaneous measurements and mathematical simulating using the instantaneous boundary conditions could find a better quantitative approximation, as the latter should be included in the evaluation because of their possible variations in longer period of time.

Content list

1. Objective of work

In order to qualify the cooling pond as an adequate thermal dissipater for the heat expelled by the station, will be necessary a composed analysis of the geographical, atmospheric, solar and water characteristics. The varying atmospheric and solar conditions make the basin to be a different characteristics dissipator everytime. Each one of the atmospheric or solar factors cannot determine by itself the dissipating capacity of the basin. All the elements are highly bound and must be treated simultaneously. The main objective is to establish these influences and to balance them with the heat coming from the nuclear power station. This analysis will provide us a base for establishing the capability of the basin to dissipate the heat completely or otherwise to calculate the net mean water temperature increase.

The water temperature at the station intake will be the solution to a dynamic flowing and heat transfer problem that may be treated with PHOENICS. We attempted to apply the CFD codes [3] in a simulation of two-phase mathematical model of the hydrothermal processes in a cooling pond as the first approximation [4], including the effects of three-dimensional (3D) structure features of the transport, power and direction of the wind, variable density of the water, and heat conduction at the water-air interface.

2. Description of the phenomenon simulated

2.1 Qualitative

This work describes the thermal evolution of a natural mass of water submitted to several environmental conditions. The length of the basin is 14.3 km, its mean width is 5.3 km and depth is from 7 to 35 m, area is 61.5 km^2. The basin domain with near land relief will be discretized in Cartesian cells and the basin contour delimited with fully blocked sells and porous regions (fig.1). The

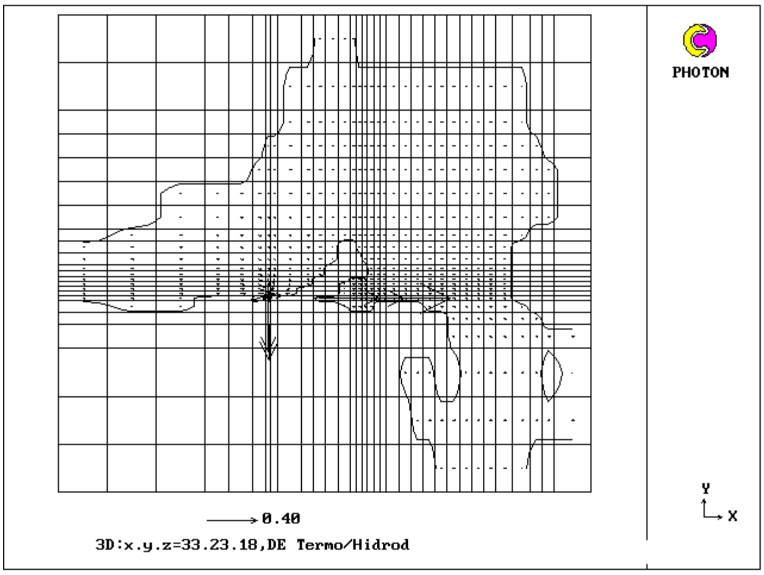


Fig.1 Difference grid (x · y · z = 33 x 23 x 18). The contours of the lake Druksiai with surface water velocity vectors near the inlet and outlet canals. The scale of velocity vectors is 0.40 m/s

grid is refined in specific zones in order to provide adequate sell size in those regions, which requires higher accuracy (i.e. near field).

The taking of water at the station intake will be considered as an outlet boundary condition of the domain where outflow occurs. It will be a geographical fixed point. The outflow of the station will be an inlet boundary condition to the domain. It will present a heat and moment source constituting one of the main factors to be considered in the thermal analysis. Finally, the set of cells half of z intervals (NZ /2 = 9, high surface), will constitute the main thermal exchange boundary condition with the solar and atmospheric environment.

2.2 Mathematical

The dimension of the computational grid, distributed by a Cartesian coordinates system, is (NX*NY*NZ = 33*23*18), that's to say, 13,662 cells although somme of them are fully blocked to the fluid and heat. It can be perceived in the fig.1 exists a refinement of the grid at the inlet boundary conditions in order to handle realistic inflow velocities in he domain. The refinement exists too at the z = NZ/2 cells.

Solution techniques and governing equations. In a general approach with recirculation of the streams and heat transfer, the problem is solved as the 3D set of the Navier-Stokes equations and energy equation for a two-phase theoretical model. The general expression is [3, 4]:

$$div\left(r_i \rho_i \vec{V}_i \Phi_i - r_i \Gamma_{\Phi i}\, grad\Phi_i\right) = r_i S_{\Phi i} \text{`}. \tag{1}$$

The set of (1) consist of differential equations with partial derivatives of Navier-Stokes and heat transfer, and are solved by the technique of finite volumes with specific boundary conditions [3, 4].

The real properties of water will be implemented for accounting the existent links between its density, heat capacity, thermal conductivity etc. and the temperature (water in liquid state). These linkings, and the establishment of conditions that simulates the action of the gravity, will establish the conditions for the possible formation of natural convection phenomenon and buoyancy forces.

The program codes used evaluate density of the water as a function of temperature [3, 5]:

$$\begin{aligned}\rho = (&999.83952 + 16.945176t - 7.9870401\times10^{-3}t^2 - 46.170461\times10^{-6}t^3 + \\ &105.56302\times10^{-9}t^4 - 280.54253\times10^{-12}t^5)/(1 + 16.879850\times10^{-3}t),\end{aligned} \tag{2}$$

where t is temperature ^{0}C. The existence of a density gradient combined with a body force as the gravity action may cause a buoyancy force responsible of a free convection phenomenon, which may be important in the fluid motion. However, it will be a good approximation to consider constant other water properties, as the thermal conductivity or heat capacity, because of less effect of their respective gradients on the fluid motion.

In the following will be described the boundary conditions that influences in the thermal and dynamic evolution of the problem.

Surface exchange. Considering adiabatic bottom and walls, the only capacity of thermal dissipation of the mass of water occurs by means of the heat exchange at the surface with the atmosphere. The main factors considered which will determine as much the global distribution of temperature as the amount of total dissipated energy. The addition of the net effects of these energetic factors will represent the amount of global energy that the basin surface is capable to dissipate.

We will study the distribution of temperature at the surface of the basin because, although the global mean temperature of the basin may increase, it doesn't mean that couldn't be appreciated certain change of the temperature at the station cooling water intake.

Forced convection exchange. The incidence of the air stream over the basin surface implicates the inclusion of a heat transfer forced convection factor. The heat transfer rate that governs this situation has the form:

$$q = h\,(T_{water} - T_{air}), \qquad (3)$$

where q represents the convective heat flow (W/m^2), which is proportional to the difference of temperatures between the water and the air. The proporcionality constant h represent all the factors that influences the convective transfer, that is to say, the conditions of the boundary layer whose characteristics depend on the contact surface geometry (waves), characteristics of the moving fluid (air), thermodynamic and transport properties (air velocity, etc.). The forced convection due to the incidence of the atmospheric air on the surface, could be represented by means of a coefficient h of the form:

$$h = \mathrm{St}\cdot\rho_a\cdot V_{rel}\cdot C_a\,, \qquad (4)$$

where ρ_a is the air density, V_{rel} is the relative velocity between the air and the water, C_a is the specific heat of the air, St is the Stanton number. In a reposed air-water interface, it has an approximated value of St $\approx$ 0.0033 and now q is:

$$q = \mathrm{St}\cdot\rho_a\cdot V_{rel}\cdot C_a\,(T_{water} - T_{air})\,. \qquad (5)$$

If we have the next values: ρ_a = 1.2 kg/m^3, C_a = 1005 J/(kg K), V_{rel} =1 m/s, then will obtain the next value of (4): h = 3.62.

Vaporization cooling. The necessary energy for vaporization should come from the internal energy of the liquid that consequently must decrease its temperature producing the consequent cooling.

Considering a stationary system, the transfer of energy from the liquid to its contours restores the latent energy lost by the water in the vaporization. This transfer will take place through all the factors of exchange that we are considering, i.e. air convection, radiation etc., and if these factors are not enough, the system will appeal to the internal energy of the water, decreasing consequently its temperature.

The heat lost by the effect of the vaporization is directly related with the mass transfer from the liquid phase to the gaseous one. This analogy it is used in order to determine the effect of the vaporization cooling at the surface.

The mass flow at the air-water interface could be expressed in the next way:

$$M = \mathrm{St}\cdot\rho_a\cdot V_{rel}\,(m_s - m_a), \qquad (6)$$

where m_s and m_a represent the mass fraction of water steam in air at the surface and in the atmosphere respectively. The mass fraction of water in the air adjacent to the surface, m_s, depends only on the temperature T_s and the water salinity, to a determined pressure. Therefore $m_s = m_s$ (T_s, salinity, P). The variation of m_s with T_s is an increasing function while decrease with salinity.

The values of m_a are obtained from the air relative humidity data of the geographical zone. The values of m_s for pure water and a pressure of one atmosphere have a dependence with the temperature of the water at the surface that we could obtain from the experimental data [3]. We can get from it a linear relation by means of the adjustment of these points to a straight line as

$$m_s = a + b\,(T_s - c), \tag{7}$$

where a =0.08205, b = 0.00115 and c = 72.5128. Consequently the mass flow M may be expressed as:

$$M = \mathrm{St}\cdot\rho_a\cdot V_{rel}\,(0.08205 + 0.00115\,(T_s - 72.5128) - m_a)\,. \tag{8}$$

The amount of transferred mass is related with the lost heat amount by means of the latent heat of vaporization of the water $L = 2.55 \times 10^6$ J/kg. The expression that connects both concepts is:

$$Q_{vap} = M\cdot L = L\cdot\mathrm{St}\cdot\rho_a\cdot V_{rel}\,(0.08205 + 0.00115\,(T_s - 72.5128) - m_a). \tag{9}$$

The mass fraction of steam in the atmosphere m_a is obtained from the relative humidity and environmental temperature following the next steps:

$$P(T) = 70.434 - \frac{7362.698}{T} + 0.006952T - 9.0\ln T; \tag{10}$$

$$p = \frac{P(T)\cdot H_r}{100}; \tag{11}$$

$$m_a = \frac{p}{1-p}\,\frac{18}{29}; \tag{12}$$

where $P(T)$ is water vapor pressure (atm) at temperature $T\,(K)$, H_γ is relative humidity (%), p is partial pressure (atm) of water at temperature $T(K)$, m_a is mass fraction water vapor in the atmosphere (kg_{H2O}/kg_{air}).

The expression (9) completed with the eq. (10) - (12) will be inserted in the program. We are interested in a linear source for its easier incorporation in the program, which is:

$$Q_g = \underbrace{-L\cdot St\cdot\rho_a\cdot V_{rel}\cdot 0.00115}_{Coefficient}\left(\underbrace{\frac{1.33972\times 10^{-3} + m_a}{0.00115}}_{Value} - T_s\right). \tag{13}$$

The solved variable will be again the water temperature T_s and the reminder magnitudes will be entering data to the program, which will be given according to the external conditions.

Incidence of direct solar radiation. The incidence of solar direct radiation over the basin surface induces the heating of the water; its value depends on its specific heat at constant pressure ($C_{p.}$ = 4183 J/(kg K), so that $Q_{sun} = C_p\,T$. The values of Q_{sun} in Ignalina (Lithuania) are picked up from the table 1.

Month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
S^I, W/m^2	70	200	240	316	392	436	402	326	275	170	100	70

Table 1. Values of the mean solar intensity on horizontal in a mean day of every month in Ignalina.

The effects of these incidences will be affected by the optical reflection over the surface, represented by a determined index (not all the incident energy is transmitted and absorbed). Only part of the net energy that impinges on the surface of the basin (table 2) is used for increasing the heat of the water. The residual energy is lost in form of reflected energy. This factor will depend on the incidence angle of the solar beam and the refraction index of the water n = 1.33 (ignoring its wavelength dependence). Both contributions are related each other by means of the Fresnel laws. The incident radiation fraction that penetrates the basin surface (P), and is absorbed by the water for its posterior transformation in heat, is expressed in the Fresnel formula:

$$P = 1 - \frac{1}{2}\left[\frac{sc^2(z-\theta_r)}{sc^2(z+\theta_r)} + \frac{tg^2(z-\theta_r)}{tg^2(z+\theta_r)}\right], \qquad (14)$$

where θ_r is refraction angle on the water surface, z is zenithal solar distance (90^0 – solar altitude). We will use too the Snell refraction law in order to determine θ_r as a function of the refraction index n and the solar zenithal distance z:

$$n\, sc\theta_r = sc\, z\,. \qquad (15)$$

Particularizing for Ignalina, the values of the monthly solar altitude has been calculated from eq. (17).

Incidence of diffuse solar radiation. The diffuse solar radiation is also an important factor since its value oscillates from a 13% of the direct radiation incidence for high solar altitude (90^0) to a 150% or rather for smaller altitudes.

On clear days could be established the next experimental table which relates the solar altitude with the direct radiation fraction that constitutes the diffuse radiation.

The data of the table 2 are fitted by means of an equation that relate the solar altitude and the percentage of direct radiation [3]. The adjustment equation that we will used in the program in order to determine the diffuse radiation, is of the form:

$$I_{solar},\% = a + b/h,\ a = 2.807,\ b = 727.1, \qquad (16)$$

$$h = \arcsin(\cos\varphi \cdot \cos\delta \cdot \cos\gamma + \sin\varphi \cdot \sin\delta). \qquad (17)$$

Solar altitude	I_{solar}, %	Solar altitude	I_{solar}, %
5	148	50	17,24
10	75,5	55	16,02
15	51,3	60	14,93
20	39,16	65	13,99
25	31,89	70	13,19
30	27,04	75	12,50
35	23,58	80	11,89
40	20,98	85	11,36
45	18,96	90	10,89

Table 2. Percentages of direct radiation which constitutes the diffuse radiation as a function of the solar altitude.

where h is solar altitude (^{0}), γ is solar hourly angle (^{0}), φ is geographical latitude (^{0}), δ is solar declination (^{0}): $\delta = 23.5 \cos(30 \cdot K - 187)$ and K is number of month set (1, 2,...,12).

Consequently, we will introduce a source of additional radiation whose value will be a determined percentage of direct incident radiation and this will establish the effect of the diffuse radiation over the basin surface.

Radiative exchange atmosphere-water and water-atmosphere. The heat exchanges by the radiation between the atmosphere and the water or vice versa is produced by the mere fact that both mean posses a different temperature of the absolute zero. This quantity of energy is interchanged and transported by means of emissions of electromagnetic nature in a band of infrared emission. The global amount of energy is obtained by integration above all the wave longitudes of the band of emission, resulting in a numerical relation that relates the energy emitted by a body as function of its temperature. This relation is the Stefan-Boltzman law and will be applied to evaluate the effect that this exchange of energy on the water temperature:

$$Q_{rad} = 4\varepsilon \cdot \sigma \left(T_{H2O}^4 - T_{air}^4 \right), \qquad (18)$$

where ε is the emissivity ($0 < \varepsilon < 1$), σ is the Boltzman constant $\sigma = 5.777 \times 10^{-8}$ W/m^{2}K^4. The emissivity ε is characteristic of each medium, and represents the effective difference of the radiant medium with a perfect black body emission at the same temperature. For Druksiai lake $\varepsilon = 0.96$.

Moment exchange between the air and the surface. A source of superficial moment will exist as a consequence of the incidence of the air on the surface of the water. This moment source will be expressed in a linear form and we will characterize it by intensity and direction. The effect of an air stream over a water surface, from a merely kinetic point of view, will be inducing on the contact surface moment source proportional to the strength of the incident wind.

We will express this effect in the program by means of the establishment of two moment sources, whose combined action will reflect the effect of the incident wind. Each one of sources will represent the incidence in a determined direction: one source will represent direction West-East, and the other one, the South-North: u – moment in West-East direction, v – moment in South-North direction:

$$\begin{aligned} S_u &= C(V - u) \\ S_v &= C(V - v) \end{aligned} \qquad (19)$$

The value of C and V will indicate the strength of the global velocity vector that represents the wind incidence.

When the simulating includes the hypothesis of turbulent viscosity, the simple viscosity factor is replaced in the transfer relations by the effective viscosity factor

$$\nu_1 = \nu_l + \nu_t \, . \qquad (20)$$

For flows of air of about 5 m/s, we used a constant factor of turbulent viscosity of 6,7, as in [5].

In our computation we divided the range of integration by a normal line. One of the parts contained 9 horizontal layers in the volume of water, and was intended for the evaluation of the geometry of the shoreline and the depth. The other part (contained 9 horizontal layers in the volume of air covered over-water flows of air and land), that is velocity, direction and force of the wind with influence of the nearest relief. The parameters of the pond, of the hot-water plume and of the cool water return must be as close as possible to the actual values. In our study we used the flow-rate of the hot-water discharge.

We present an analysis of the effect of a weak wind (0 -5 m/s) on the hydrothermal behavior in the pond, for next values of turbulence in the air:

$$\nu_t = 1{,}34 \times \left| U_{loc} \right| \qquad (21)$$

and mixing in the water according [3, 4].

3. PHOENICS settings

All settings are made using a Q1 file with calls to the GROUND subroutine for all non-standard computation. Extracts from both the Q1 file and GRAUND subroutine appear in Appendices 1 and 2.

4. Results and consideration

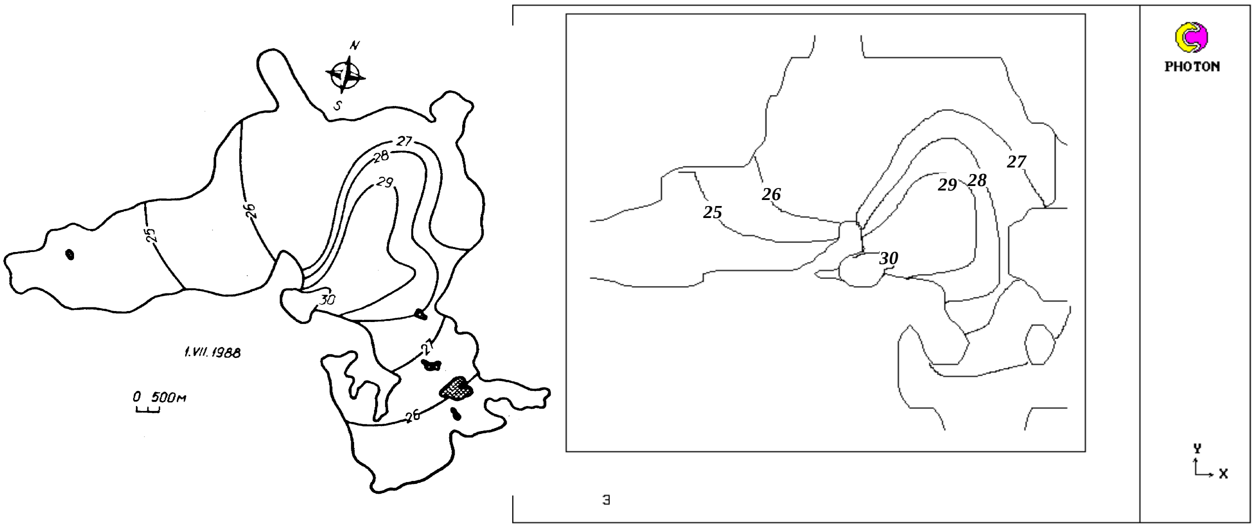
Therefore a variable-step grid was constructed, Fig 1. It covers only a certain part of the surface and the range of integration with respect to the normal covers a 10 m layer of water and a 100 m layer of air including nearest relief (with blocking cells).

In this case we evaluated the effect of the wind on the hydrothermal behavior of the lake, which come into the play by influence on the distribution of isotherms. The technique of testing was applied from [2]. There were evaluated the temperature dependence of water density, the water-air heat conduction, and the resistance factor of the water.

A numerical simulation of 33 x 23 x 18 grid system consumes a lot of computer time, therefore the computation was suspended, whenever a specific effect of wind became evident, or similar to measured data at the hot-water discharge. A closer agreement was found for the case of no wind. The Fig.2a presents the simulated results.

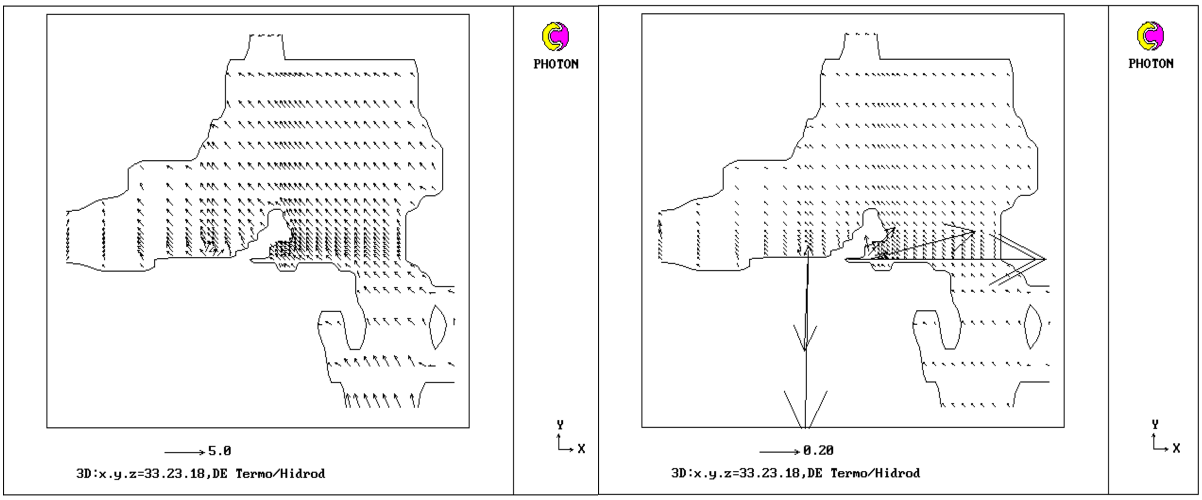
a **b**

Fig 2. Temperature distribution on the water surface of the lake Druksiai for wind velocity - 0 m/s): a - measured isotherms [2], b - predicted isotherms



a **b**

Fig 3. Temperature distribution on the surface of the lake Druksiai for average southeastern wind velocity of 1.0 m/s: a - measured isotherms [2], b - predicted isotherms. The air temperature is 23.9^{0}C.



a **b**

Fig 4. Movement of fluids at South-East mean wind velocity 1 m/s: a - horizontal vectors field in the air area 0.25 m above the water surface (vectors reference scale - 5 m/s), b - water surface velocity vectors (reference scale – 0.2 m/s)

The Fig.3 presents the predicted isotherms for a 1 m/s southwest wind. But these predictions disagree with Fig.2, because of the inexact choice of hydrodynamic conditions on the inlet. The Fig.4a and 4b shows the water surface velocity vectors distribution and wind velocity vectors field under the water surface respectively.

Predicting the effect of wind to the water surface, we used a constant water heat transfer coefficient. Mixing of the hot and cool water in the lake is a complicated process, and the wind makes it even more such, as it introduces waves and variable depths. The predictions cannot be in a good agreement with the measured results, because the wind blowing directions are variable and the actual hydrothermal state of a pond is a continuously varying behavior. This is a reason for variable water heat transfer coefficient to introduce into the future predictions.

5. Conclusions

The earlier code of numerical elliptic equations was used to construct a primary numerical model of hydrothermal dynamics in lake Druksiai. The CFD codes were applied for the numerical solution of two-phase 3D mathematical model of the flow. The solutions can evaluate the effect of wind, of temperature-dependent water density, of water-air heat conduction, of air turbulence and of the ground geometry.

Computational procedure include direct and diffusive solar radiation, radiative exchange between atmosphere and water, evaporation cooling function factor, but its influence on heat balance numerically not established.

An analysis of the numerical solutions for the hydrothermal processes in lake Druksiai, and their comparison with the test points suggest an influence of the wind, of the water-air heat conductivity, of the variable density of water, of water mixing, and partially of the geometry of the shore-line on the results of simulation, which are qualitatively similar to the test points. To approach the prediction to the actual state, the possible time-dependent set equations and boundary conditions should be used.

6. Nomenclature

r_i - volume part of phase i; ρ_i - density of phase i, kg/m^3;
Φ_i - dependent variable of phase i: 1 for continuity eq., U, V, W impulse in directions x, y and z respectively, m/s, H enthalpy (temperature);
$\vec{V}_i$ - velocity vector of phase i;
Γ_Φ - exchange coefficient of variable Φ;
S_Φ - source term in the flow for variable Φ;
p - pressure N/m^2;
U_o - inflow velocity, m/s;
u' - longitudinal component of fluctuating velocity, m/s;
x, y, z - Cartesian co-ordinates, m;
ν - kinematic viscosity (ν_l – laminar, ν_t - turbulent), m^2/s

7. Literature References

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